

$V(J, P)$	the J th velocity mode in the material at pressure P
$\tau(J, P)$	the travel-time for the J th velocity mode at pressure P
$F(I, J, P)$	the I th null frequency observed for the J th velocity mode in the material at pressure P
$N(I, J, P)$	the number of $\frac{1}{2}$ wavelengths in the specimen corresponding to $F(I, J, P)$
$\tau(I, J, P)$	the travel time in the specimen corresponding to $F(I, J, P)$
$IMP(J, P)$	mechanical impedance of quartz transducer for J th velocity mode at pressure P
$K(I, J, P)$	$IMP(J, P)/(\text{mechanical impedance of the material corresponding to } \tau(I, J, P))$
$V(1, P)$	longitudinal velocity in the (100) direction at pressure P
$V(2, P)$	shear velocity in the (100) direction at pressure P
$V(3, P)$	longitudinal velocity in the (110) direction at pressure P

We need only know any three independent velocity modes in order to obtain the three elastic constants of a solid. In this paper the resonant frequencies measured as a function of pressure for the longitudinal modes of propagation in the (100) and (110) directions and the shear mode of propagation in the (100) direction have been used.³

We also assume the following:

- (i) The temperature dependence of the volume, or the linear expansion coefficient at a temperature T and one atmosphere is known;
- (ii) the specific heat at temperature T and one atmosphere is known; and
- (iii) $[\partial\beta(P)/\partial T]_P \approx [\partial\beta(P_1)/\partial T]_{P_1}$, where $P \geq P_1$, holds.⁴

Then the procedure outlined below can be used to estimate the elastic constants of solids at higher pressures, without reference to *a priori* knowledge of the compressibility of the substance.

The relation between the adiabatic bulk modulus and $V^2(J, P)$, ($J=1, 3$), in a cubic solid may be written as $B^S(P) = \frac{1}{3}\rho(P)[4V^2(3, P) - 4V^2(2, P) - V^2(1, P)]$. (2)

Relation (1), expressed in terms of $L(J, P_1)$, $\tau(J, P)$, $\lambda(P)$, and $\rho(P_1)$, is given as relation (3):

$$B^S(P) = \frac{1}{3}\rho(P_1)\lambda(P)[4L^2(3, P_1)/\tau^2(3, P) - 4L^2(2, P_1)/\tau^2(2, P) - L^2(1, P_1)/\tau^2(1, P)], \quad (3)$$

where $\rho(P) = \lambda^3(P)\rho(P_1)$. By the definition of isothermal bulk modulus we obtain

$$B^T(P) = -\text{Vol.}(P)[\partial P/\partial \text{Vol.}(P)]_T = \rho(P)[\partial P/\partial \rho(P)]_T = \frac{1}{3}\lambda(P)[\partial P/\partial \lambda(P)]_T. \quad (4)$$

And if

$$\Delta(P) = \beta^2(P)B^S(P)T/\rho(P)C_P(P) \quad (5)$$

where temperature T is in Kelvin, then

$$B^T(P) = B^S(P)/[1 + \Delta(P)]. \quad (6)$$

Using Williams and Lamb's⁵ method of ultrasonic wave velocity measurements as modified by Colvin,⁶ transit time for the various wave propagations is obtained from the following relations:

$$N(I, J, P) = \text{Integer}\{[F(I, J, P)/\Delta F(I, J, P)] - 0.5 - K(I, J, P)\}, \quad (7)$$

$$\tau(I, J, P) = [N(I, J, P) + 0.5]/2F(I, J, P) - [K(I, J, P)/2]\{[1/F(R, J, P)] - [1/F(I, J, P)]\}. \quad (8)$$

In the above expressions $K(I, J, P)$ may be written as

$$K(I, J, P) = IMP(J, P)/\rho(P)V(J, P) = IMP(J, P)\tau(J, P)/\rho(P_1)\lambda^2(P)L(J, P_1) \quad (9)$$

where $IMP(J, P)$ is the mechanical impedance of the transducer for the J th velocity mode at pressure P .

It is evident from relation (8) that if the measurements are made near $F(R, J, P)$ any error in the estimation of $\tau(I, J, P)$ due to inaccurate knowledge of $K(I, J, P)$ becomes negligible.

By integrating relation (4) we obtain

$$\lambda(P) = \lambda(P_1) \exp[(P - P_1)/3B^T(P)]. \quad (10)$$

Two things should be noted regarding the derivation of (10) from (4): (i) In the definition of isothermal bulk modulus at a pressure P , one could obtain its value by either decreasing or increasing the pressure slightly; and (ii) when integrating (4) it must be remembered that it is implied in the definition of $B^T(P)$ that it remains constant over the range of integration P_1 to P . In expression (10) it is implied that the isothermal bulk modulus of a substance at pressure P has been obtained by decreasing the pressure from P to P_1 . The expression for $\lambda(P)$ as derived above differs from that obtained by following either Cook's or Ruoff's procedures. The expression for $\lambda(P)$ that will be obtained by following Cook's or Ruoff's procedure may be given by

$$\lambda(P) = 1 + [\rho(P_1)L^2(1)]^{-1} \int_1^P [1 + \Delta(P)] \times \{[4/\tau^2(3, P)] - [4/\tau^2(2, P)] - [1/\tau^2(1, P)]\}^{-1} dP, \quad (11)$$